**A Defense of the Kripkean Account of Logical Truth in First-Order Modal Logic**

## 1. Introduction

The concern here is criticism of the Kripkean representation of modal, logical truth as truth at the actual-world element of every subset of the set of possible worlds. The crux of the criticism is variability from one model structure to another of the collection of worlds used in fixing the extension of logical truth for a modal language. With respect to the reading of the modal operators as the logical modality operators, this feature has been called curious, completely lacking in motivation, and held responsible for Kripkean semantics missing some logical truths. In particular, the Kripke account is criticized for failing to reflect the idea that true statements about what is logically possible are logically necessary. Critics include Pollock (1966), Cocchiarella (1975a) and (1975b), Field (1989), Hintikka (1980), (1982), and (1995), and Hanson and Hawthorne (1985).

The aim of this paper is to respond to this criticism, largely ignored in the literature, and defend the use of model structures with arbitrary sets of possible worlds in fixing the extension of modal, logical truth. Specifically, I highlight the standard semantic treatment of the modal operators as quantifiers ranging over collections of possible worlds, and argue that the modal operators should be allowed to range over various collections of possible worlds in determining modal, logical truth given that the domain of first-order quantifiers may vary in determining classical logical truth. As I explain below, the lack of a plausible rationale for allowing the yield of logical truth in first-order modal logic to turn on one reading of the modal operators understood in terms of a specific collection of possible worlds leads me to question the idea, essential to the criticism of the Kripke account, that all true sentences about what is logically possible are logically necessary.

In what follows, I first review the Kripke characterization of logical truth in first-order modal logic. Next, I develop the above criticism and sketch my defense of the Kripke account. Finally, I consider an objection to my defense which points to a disanalogy between the semantic treatments of modal operators and first-order quantifiers that is taken to undermine the relevance of variations in the set of possible worlds as representations of logically possible situations. I find the objection problematic, because I find the account of logical possibility it requires inadequate in the manner spelled out below.

## 2. The Kripkean Account of Modal Logical Truth

Let L be an uninterpreted first-order modal language. A Kripkean *semantic framework* for first-order modal languages is characterized by the use of a quantificational model structure. We may think of a quantificational model structure as a reading or interpretation of modal operators, which includes a specification of the following.

(1) A collection **K** of possible worlds

(2) Which member of **K** represents the actual world **G**

(3) An accessibility relation **R** on **K**

(4) The set $\Psi$(W) of individuals existing in each world W

(5) The set **U** of possible objects

To give a complete interpretation of L, one must give in addition a model and an assignment function that specifies extensions for the predicates, individual constants if any, and variables of L.

A model **M** is the binary function $\Phi_M(P^n,\mathbf{W})$ defined on a model structure. Informally, a model is a function that fixes the extensions of predicates in worlds, and a variable assignment assigns elements from **U** to the variables of the language. We develop this formally as follows. A model **M** is the pair $\langle \mathbf{M^*}, \Phi_M(P^n, \mathbf{W})\rangle$; **M*** ranges over quantificational model structures, $P^n$ ranges over *n*-adic L-predicates letters for arbitrary *n*, and, as above, **W** ranges over members of **K**. The function $\Phi_M$ assigns subsets of $\mathbf{U}^n$ ($\mathbf{U}^n$ is the *n*th Cartesian product of **U** with itself) as the extensions of the *n*-place predicates in L. Kripke only requires $\Phi_M(P^n, \mathbf{W}) \subseteq \mathbf{U}^n$, rather than

$\Phi_M(P^n, \mathbf{W}) \subseteq \Psi(\mathbf{W})^n$. So, since subsets of $\mathbf{U}^n$ play the role of extensions of $P^n$, the extension assigned to a predicate in a given world **W** may contain an object not in **W**. We define "truth in a model **M** based on a model structure **M***" by first defining inductively, for every L-formula $\alpha$ and every world $\mathbf{W} \varepsilon \mathbf{K}$, a value for $\Phi_M(\alpha, \mathbf{W})$ relative to an assignment *f* of elements from **U** to the variables of L. We assume that this has been done.

A formula $\alpha$ from L is *true in a model **M*** relative to an assignment *f* to the variables of L iff $\alpha$ is true in the actual world **G** of **M** relative to *f*. For $\alpha$ with no free variables, $\alpha$ is *true in a model **M*** iff for every assignment *f*, $\alpha$ is true in the actual world **G** of **M** relative to *f*. Let **S** range over classes of model structures (i.e., **S** ranges over the class of T model structures, the class of B model structures, etc.). For any **S**, we say that sentence $\alpha$ is *true in a model structure* $\mathbf{M^*} \in \mathbf{S}$ iff $\alpha$ is true in every model **M** based on **M***. A sentence $\alpha$ is *universally valid* in **S** iff $\alpha$ is true in every model structure $\mathbf{M^*} \in \mathbf{S}$.

In the balance of this paper, we focus on the use of Kripke's account of universal validity to fix the extension of logical truth for first-order modal languages adequate for thinking about specifically logical modality. Since logical modality is generally taken to be an S5 modality, we focus on the class of S5 model structures and take the box and the diamond of language L to be the S5 operators which represent the usual, intuitive, meta-linguistic notions of logical necessity and possibility for first-order sentences.[1]

To illustrate the Kripke account in action, consider two interpreted modal sentences.

(A) $\Diamond_{S5}\exists x$ (Female (x) & U.S. President (x))

(B) $\Diamond_{S5}\exists x\exists y(x\neq y)$

Both (A) and (B) are true. Consider the model structure **M***: $\mathbf{K}=\{\mathbf{G}\}$, the accessibility relation **R** is an equivalence relation, $\mathbf{U}=\{\mathbf{u_1}\}$ and $\Psi(\mathbf{G})=\{\mathbf{u_1}\}$. Now we define a model **M** on the above model structure **M*** as follows: $\Phi_M$(Female, **G**) = $\{\mathbf{u_1}\}$, and $\Phi_M$(U.S. President, **G**) = $\varnothing$. Both

(A) and (B) are false in **M**.[2] So, if *universal validity* relative to the class of S5 structures is logical truth in S5 first-order modal logic, then neither (A) nor (B) is a logical truth.

Obviously, the model structure **M*** differs from what it should be given the intended reading of '$\Diamond_{S5}$' in (A) and (B) as the logical possibility operator. For example, **K** consists of just one possible world, but there are more logically possible worlds. So, the Kripke account judges that (A) and (B) could logically be false by appealing to a model structure that does not portray logical modal reality. In essence, Kripke S5 model structures may contain distinct non-empty sets of "all" logically possible worlds.[3] Kripke remarks that,

> In trying to construct a definition of universal logical validity, it seems plausible to assume not only that the universe of discourse may contain an arbitrary number of elements and that the predicates may be assigned any given interpretation in the actual world, but also that any combination of possible worlds may be associated with the real world with respect to some group of predicates. In other words, *it is plausible to assume that no further restrictions need be placed on [**U**], **G**, and **K**, except the standard one that [**U**] be nonempty* [italics mine]. This assumption leads directly to our definition of universal validity. (Kripke (1959), p. 3)

However, Kripke does not motivate the use of universal validity as a characterization of logical truth in first-order modal logic. Indeed, not only is the plausibility of the assumption that no further restrictions need be placed on **U**, **G**, and **K** (except the standard one that **U** be nonempty) questioned, but there is substantial criticism of this assumption in the context of the application of universal validity to fixing the extension of logical truth for an interpreted modal language in which the modal operators represent logical necessity and possibility.

## 3. The Criticism

Critics have held that determining what is universally valid is irrelevant to the systematization of what is logically true when the diamond and box represent the logical modalities.[4] *Universal validity* seems applicable as a characterization of logical truth in the logic of physical possibility. Clearly, there are different logically possible ways that the set of all physically possible worlds might have been. But it does not seem logically possible that the class of all logically possible worlds be different in the ways required by Kripkean semantics. For example, it does not seem logically possible that, say, the class of all logically possible worlds consist of just one world, as the above countermodel to (A) and (B) would have it. Logical facts can't be otherwise; they are not contingent. According to the criticism, an adequate logic of logical modality must reflect the following *thesis about logical possibility* (TLP):

All true statements about what is logically possible in first-order logic are logical truths.

The formal representation of (TLP) is the S5 characteristic axiom $\ulcorner\Diamond_{S5}\alpha\rightarrow\Box_{S5}\Diamond_{S5}\alpha\urcorner$. Hence, the choice of S5 as the correct logic of logical modality is essential to the criticism of the Kripke account. Accordingly, the criticism may be put as follows. An adequate account of logical truth for language L must make it the case that for arbitrary non-modal L-sentences $\alpha$, if $\ulcorner\Diamond_{S5}\alpha\urcorner$ is true, then $\ulcorner\Diamond_{S5}\alpha\urcorner$ is logically true. (TLP) fails on the Kripkean account (see Lindström (1998) p. 230, who argues against the Kripkean account on the basis of (TLP)) because on the Kripkean account, $\ulcorner\Diamond_{S5}\alpha\urcorner$ is a logical truth (because it is universally valid), only if $\alpha$ is a first-order logical truth. Again, let $\alpha$ be any non-modal first-order sentence. $\ulcorner\Diamond_{S5}\alpha\urcorner$ is universally valid only if for all models **M** defined on an S5 structure **M*** according to which **K** is restricted to {**G**}, $\Phi_M(\Diamond_{S5}\alpha, \mathbf{G})=\mathbf{T}$. But such a model requires $\Phi_M(\alpha, \mathbf{G})=\mathbf{T}$. Since any $\mathbf{W}\in\mathbf{K}$ world can play the role of **G** in **M*,** $\alpha$ must be true at all logically possible worlds, i.e. $\alpha$ itself must be a first-order logical truth. In short, since the Kripke account does not reflect (TLP), it is reasonable to think that it is inaccurate.

What the critics advocate is that the logical truth of $\ulcorner\Diamond_{S5}\alpha\urcorner$ should boil down to the truth of $\ulcorner\Diamond_{S5}\alpha\urcorner$ at the actual-world element **G** of each model defined on the S5 structure *as fixed by the correct representation of logical modal reality*. We should use *truth in a model structure **M**** to represent logical truth, where **M*** is the course-grained representation of the modal reality privileged by the intended reading of the diamond and box as the logical possibility and necessity operators. In terms of the Kripkean semantic framework, from one model to the next, **U** should just consist of all the logically possible objects, **G** should be the logically possible world that is actual, and **K** should correspond to all of the logically possible worlds.

The criticism requires what I call a fixed modal operator semantics. A fixed modal operator semantics is any modal semantics that relativizes logical truth to one reading of the modal operators as this is given by (1)-(5) on p. 1. According to a fixed modal operator semantics, a modal sentence *p* is a logical truth iff *p* is true in each world in the one quantified model structure given by the intended sense of the modal operator(s) occurring in *p*. Recall that by the lights of Kripke's account, we may ignore the intended sense of the modal operator(s) appearing in *p* and show that *p* is not a logical truth by varying (1), (2), and (5).[5]

There are two types of defenses of the Kripke account that one can give in response to the above criticism: a pluralistic and a non-pluralistic defense. Each defense uniquely characterizes what is at stake in the debate over the legitimacy of the Kripke account. A pluralistic defense argues that since the aim of the Kripke account is to capture logical truth *qua* truth in *every* S5 model structure while the aim of a fixed modal operator account is (in terms of the Kripke semantic framework) to capture logical truth *qua* truth in the *one* S5 model structure determined by the use of the S5 box and diamond as the logical modality operators, it is confused to fault the Kripke-type account for failing to realize the aim of the non-Kripkean, fixed modal operator account. Since the fixed modal operator and the Kripke-type accounts are after different things (logical truth *qua truth in a structure* and logical truth *qua universal validity*), the fact that the accounts differ extensionally does not suggest that the latter account is deficient.[6] It is confused to criticize the Kripke account on the basis that "the so called Kripke semantics does not provide us with the right model theory of logical (conceptual) necessities." [7] This is criticizing apples for not being oranges.

This defense of the Kripke account does nothing to undermine the claims made by the discontents that a fixed modal operator semantics is more appropriate than Kripke semantics when one is concerned with the logic of specific modal notions like logical necessity and possibility. So on a pluralistic defense of the Kripke account, (TLP) and the extensional correctness of the fixed modal operator account of the logic of logical modality are not at issue.

The pluralistic defense of the Kripke account of logical truth eschews a pre-theoretical notion of logical truth according to which one may criticize (TLP) and fixed modal operator semantics. Whether or not sentences (A) and (B)

(A) $\Diamond_{S5}\exists x$ (Female (x) & U.S. President (x))
(B) $\Diamond_{S5}\exists x\exists y(x\neq y)$

are logical truths depends on whether we understand logical truth as universal validity or in the fixed-modal-operator way. The matter is left here on a pluralistic defense of the Kripke account.

What I am calling the pluralistic defense views the Kripke account as providing a single logic-of-modality for the S5 modal operators that obeys only the rules that all of the more specific fixed S5-possibility operators (e.g., logical possibility, physical possibility, and all-sorts-of other-S5-possibility operators) have in common. This approach has been discussed in the literature and rejected by at least some of the critics (e.g., Hanson and Hawthorne). In the opinion of these critics, a single logic-of-modality for the S5 modal operators is of no more interest than a "variable" connective whose logic incorporates only those features that all truth-functional binary connectives have in common, or a "variable" quantifier whose logic incorporates only those features that all unary individual quantifiers have in common. This motivates interest in a non-pluralistic defense of the Kripke account.

In contrast to a pluralistic defense, a non-pluralistic defense aims to show that the Kripke account is correct even when the modal operators are interpreted unambiguously as expressing logical modalities. This defense argues that the Kripke account is the right semantic account of the specific modality of first-order logical necessity by calling into question the truth of (TLP). That is, a non-pluralistic defense of the Kripke account argues that the S5 characteristic axiom does not represent a truth about logical modality. According to the concept of logical possibility, properly understood, sentences such as (A) and (B) are not logical truths because it is logically possible for them to be false. Below I shall pursue a non-pluralistic defense of the Kripke account.

## 4. A defense of the Kripke Account of logical truth in modal first-order logic

My mainline defense of the Kripke account is the following argument for thinking that (TLP) is false.

1) The standard account of how quantifiers work in first-order logic is as follows: a quantified sentence is a logical truth only if it remains true no matter how the quantifiers are restricted (and no matter how the interpretations of non-logical expressions are varied).
2) The natural extension of this to the semantics for the modal operators—which on many views are analogous to the quantifiers—is to say that a modal sentence is a logical truth only if it remains true no matter how these "quantifiers" are restricted (i.e., no matter what subset of logical space we have them range over).
3) (TLP) is false, i.e., some true statements about what is logically possible are not logical truths.

In first-order logic, the range of quantifiers varies from interpretation to interpretation. For example, to show that '$\exists x \exists y \forall z(x \neq y \ \& \ (z=x \vee z=y))$' is logically possible we invoke an interpretation whose domain has only two elements. The sentence is false on any interpretation with more than two elements. What do we get when we extend this to modal logic? Thanks in large part to Kripke, the semantic treatment of the box and diamond as quantifiers over possible worlds is standard. To pursue the analogy with the semantic treatment of ordinary quantifiers, in every model, '$\Diamond_{S5}$' means 'in some possible world', and the range of possible worlds varies from model to model. But then we have a counterexample to (TLP). (B) '$\Diamond_{S5}\exists x \exists y(x \neq y)$' although true, is not logically necessary for it is false in any model according to which the range of '$\Diamond_{S5}$' is restricted to those worlds in which there is just one element.

The argument poses a challenge to the proponent of fixed modal operator semantics. If the standard definition of logical truth for a non-modal first-order language considers many different ranges for the individual quantifiers, then why shouldn't the definition of logical truth for a modal language consider many different ranges (i.e., many different model structures with different sets of possible worlds) for the modal operators? Before entertaining an objection to my argument against (TLP), I sharpen the philosophical picture behind the argument.

The standard account of how quantifiers work in first-order logic is desirable because, in part, it captures the idea that a logically true quantificational sentence is true in virtue of form. According to this notion of truth in virtue of form,

if $\phi$ is non-modal and $\psi$ is obtained from it either by substituting formulas for non-logical predicates, or by uniformly restricting the ranges of all quantifiers and free variables, or both, then if $\phi$ is logically true, so is $\psi$. [8]

Following Field, let's call this the logical form principle (LFP). Note two features of logical truth highlighted by (LFP). First, a characterization of logical necessity applied to first-order sentences that obeys (LFP) must be closed under substitution. This is a relatively weak requirement on what counts as a logical truth. Classical, intuitionist, mainstream modal, relevance, tense, etc. logicians—i.e., Frege, Russell, Heyting, C.I.Lewis, Lukasiewicz, Anderson-Belnap, Prior, *et al*.—have all understood logical truth to be closed under substitution. Second, according to (LFP), if a first-order quantification is logically true, then it remains true regardless of the non-empty subset of the world's individuals that serves as the domain of discourse. By (LFP), the logical form of a universal or existential quantification is fixed independently of the intended domain of discourse, i.e., the intended domain is a non-logical component of the quantifier. So (LFP) does not require that the instances of a quantificational form have the same domain. For example, 'there are at least two things', 'there are at least two natural numbers', 'there are at least two Republicans who are pro-choice….' can reasonably be viewed as instances of '$\exists x \exists y (x \neq y)$' where the domains of the existential quantifiers are the set of first-order particulars, natural numbers, and Republicans who are pro-choice, respectively.

On my view, the intuitive, pre-theoretic understanding of logical possibility that steers our technical characterization of logical truth in extensional logic should be in sync with the technical characterization of logical truth in modal logic. As Pollock remarks, "…a concept of semantical validity which adequately explains the concept of logical validity must be applicable in general to any system of logic—higher order logic, modal logic, etc.—and not just to the elementary predicate calculus."[9] Turning to modal logic, the semantic treatment of '$\Box_{S5}$' and '$\Diamond_{S5}$' as a universal and an existential quantifier is standard. The extension of (LFP) to modal logic is in the form of (LFP').

(LFP ')

If $\phi$ is a modal sentence and $\psi$ is obtained from it either by the substitution of formulas for non-logical predicates, or by uniform non-empty restrictions of the ranges of all modal quantifiers then if $\phi$ is logically true, so is $\psi$.[10]

(LFP') reflects the requirement that a modal sentence is logically true only if it remains true on *any assumption about the non-empty range of the modal quantifiers '$\Diamond_{S5}$' and '$\Box_{S5}$'* and no matter how the interpretations of non-logical expressions are varied. On my view, some principle such as (LFP') is necessary in order to represent the formality of modal, logical truth. It insures that modal, logical truth is closed under substitution and is invariant across the range of assumptions about the non-empty domain of the modal quantifiers '$\Diamond_{S5}$' and '$\Box_{S5}$' i.e., in assigning a truth value to a modal sentence, the range of the S5 modal operator(s) can be any non-empty subset of the totality of possible worlds in a S5 model structure.

Clearly, (LFP') conflicts with (TLP), and, therefore, militates against pursuing the logic of logical modality in an S5 setting. For example, (A) '$\Diamond_{S5}\exists x$ (Female(x) & U.S. President(x))' is a fixed modal operator logical truth despite the fact that (A) becomes false upon substituting 'Non-Female' for 'U.S. President'.[11] (B) '$\Diamond_{S5}\exists x\exists y(x\neq y)$' is regarded as a logical truth by the fixed modal operator theorist, even though (B) becomes false when we let '$\Diamond_{S5}$' range over just one logically possible world whose domain is exactly one individual. Where '$\Box_L$' is the logical necessity operator, we want both $\ulcorner\Box_L\alpha\rightarrow\alpha\urcorner$ and $\ulcorner\Box_L(\alpha\rightarrow\beta)\rightarrow(\Box_L\alpha\rightarrow\Box_L\beta)\urcorner$ to hold for all sentences $\alpha$, $\beta$, Also, we want $\ulcorner \Box_L\alpha \urcorner$ to turn out logically necessary whenever $\alpha$ is logically necessary and take $\ulcorner \Box_L\alpha \rightarrow \Box_L\Box_L\alpha \urcorner$ to hold. These principles of '$\Box_L$' are the S4 axioms for the necessity operator. On my view, the correct logic of logical modality is at least as strong as S4, but not as strong as S5.

According to the fixed modal operator theorist, sentences (A) and (B) are logically necessary, and she takes this to represent that what (A) and (B) *mean* is logically necessary (i.e., the state of affairs they report couldn't logically be otherwise). However, when I say that (A) and (B) could logically be false (again, reading the S5 diamond as the logical possibility operator) what I mean is that their logical forms have false instances. I accept that (A) and (B) are truths of logic. The defenders of (TLP) parse "truth of logic" as "logical truth", which doesn't seem promising. Any statement of a fact about logic might qualify as a truth of logic, but few of them are logical truths.[12] For example, the completeness and soundness theorems for first-order logic express necessary facts about logic, but they are not true in virtue of form, and, therefore, they are not logical truths. Of course, it is possible to construe the logical forms of (A) and (B) so that they are true solely in virtue of form. However, the rationale for doing this is unclear given my argument against (TLP). Is the argument cogent?

The most promising criticism is to attack the second premise by pointing to significant differences between modal operators and individual quantifiers, and then show how these differences justify a fixed modal operator semantics for modal languages. We say that first-order models represent logically possible states of affairs (or at least the structures of logically possible states of affairs). The reason that the domains of first-order models vary is that for each natural number *n* there is a logically possible state of affairs according to which there are just *n* individuals. Hence, in fixing the extension of logical truth for first-order quantifications, we should take into account different quantifier ranges. The situation with respect to the operators for logical modalities is significantly different. There are many logically possible worlds, and the operators for logical modalities must quantify over all of them for there is no logically possible state of affairs according to which there are just *n* logically possible worlds for some finite natural number *n*.

On my view, the success of this criticism turns on the plausibility of an account of logical possibility according to which (C)-(E) are true.

(C) For each natural number *n* there is a logically possible state of affairs according to which there are just *n* individuals.

(D) It is metaphysically necessary that there are at least infinitely many individuals.

(E) There is no logically possible state of affairs according to which there are fewer logically possible worlds than there are.

Obviously, the criticism requires (C) and (E). (C) grounds *quantifier range restrictions* in first-order logic. (E) provides the rationale for ignoring Kripke models. I hold (D), which I will not defend here, and so on my view there is some work to do in order to secure (C). Specifically, we expect an account that uncovers the nature of logical possibility to explain how it is possible for there to be just finitely many individuals even though this is metaphysically impossible.

I say that the above criticism of my argument against (TLP) fails because there is no plausible account of logical possibility that secures (C)-(E). In order to illustrate the challenge of producing the required story of logical possibility, I shall sketch three accounts of logical possibility that are in the literature. I will assess each in terms of its capacity to respect (C), (D) and (E), highlighting how it handles quantifier range restrictions in first-order logic. To the best of my knowledge, the three accounts jointly offer the only rationales consistent with (D) for (C).

## 5. Three Accounts of Logical Possibility

### (I) *Logical Primitivist Account of Logical Possibility*

(C) A logically possible state of affairs according to which there are just *n* individuals is a logically possible world with just that many individuals. A logically possible world is one way the world could logically be.[13] For each finite *n*, there is a logically possible world with just *n* individuals because it is logically possible that the world contain just *n* individuals. So different first-order quantifier ranges reflect the logical possibility of the world containing different numbers of individuals.

(D) In order to get (C) in sync with (D), the logical primitivist admits a special logical modality. Logical possibility is primitive. What is logically possible does not supervene on what is possible in any other sense. For example, there may be no metaphysically possible world with just one individual, but a Parmenedian world is a logically possible one. So, while the sentence, "there are at least two individuals" expresses a metaphysically necessary truth, it is logically possible for the sentence to be false. Belief in the possibility of a Parmenidean world is justified on the basis of brute logical intuition, even though science, mathematics and metaphysics may rule it out.

(E) A logically possible state of affairs according to which there are just *n* logically possible worlds for finite *n* is a logically possible modal reality with just *n* logically possible worlds. A logically possible modal reality is one way modal reality could logically be.[14] The only rationale for different ranges for the modal "quantifiers" would be to reflect the logical possibility of modal reality containing different numbers of logically possible worlds. However, by (C), there are many logically possible worlds and the operators for logical modalities must quantify over all of them for there is no logical possibility of there having been different numbers of logically possible worlds.

Criticism of my defense of the Kripkean account that relies on logical primitivism assumes the burden of defending this approach to logical possibility. While logical primitivists offer different rationales for treating logical possibility as basic,[15] many theorists are unwilling to regard logical possibility as a primitive notion.[16] One may be dissatisfied with leaving logical possibility unexplained. As Shapiro remarks, "accept a primitive notion of possibility, and we are left with very little idea of what this notion comes to."[17] Logical primitivists owe us an account of how we could come to understand what is logically possible, as we in fact do, independently of our science, mathematics and metaphysics. In addition, it is desirable that we secure simpler explanations of logical possibility, in particular ones that do not grant a special logical modality, while handling traditional assumptions of first-order logical possibility such as (C). Critics of the Kripkean account who are not logical primitivists will be motivated to look elsewhere for the needed account of logical possibility.

II. *A Non-modal Account of Logical Possibility*[18]

(C) A logically possible state of affairs is either a metaphysically possible world or a fragment of one. A first-order model with a restricted domain represents a fragment of a metaphysically possible world. On this approach, a sentence is logically possible iff there is a metaphysically possible world or a fragment of one according to which the sentence is true on some meaning assignment to the sentence's non-logical terminology. Borrowing from an example of Hanson's, if we restrict our attention to, say, the Washington Monument and the White House, we have a logically possible state of affairs (a fragment of the actual world and so of a metaphysically possible world) according to which '$\exists x \exists y \exists z\ (x \neq y\ \&\ y \neq z\ \&\ x \neq z)$' is false.

Fine (2002) regards logical necessity as a species of metaphysical necessity. He suggests that we say that a logical necessity is a metaphysical necessity that is a *logical truth*, and then define *logical truth* non-modally. I believe that what I am calling the non-modal approach to quantifier range restrictions reflects Fine's suggestion. For we may derive the following characterization of logical truth from the non-modal account of logical possibility: a sentence $\alpha$ is a logical truth iff for each metaphysically possible world $w$, $\alpha$ is true in $w$ and true in *every fragment* of $w$ under *every interpretation* of its non-logical terminology. The italicized portions are non-modal generalizations.

(D) Here, unlike on the logical primitivist approach, models with restricted domains do not represent full-fledged possibilities and so (D) is respected. According to Hanson, the rationale for admitting models with restricted domains is that, while not representing genuine possibilities, they should be considered "as sub-worlds of possible worlds as a legitimate way of ensuring that logic exhibits an appropriate generality."[19] Hanson remarks that

A mathematical platonist who adopts this approach is claiming that something stronger than necessary truth is required in an adequate account of logical consequence, but this needn't be seen as appealing to a special logical modality. Instead it can be seen as claiming for the sake of generality, one is willing to accept as counterexamples to arguments not only full-fledged ways things might have been but also fragments of such ways. I believe that logicians often think of logical models in this way, and that doing so does not commit them to the view that each such model is itself a full-fledged possible world (that is, a way things might have been). Doing so can be seen as introducing further generality into logic, a generality that comes from taking logic to be applicable even to the tiniest and most bizarrely gerrymandered fragments of possible worlds.[20]

The generality requirement motivates the formal criterion of logical truth: by considering all the sentences that have a certain property in virtue of form we achieve a generality that is hard to come by. Also this explains why logical constants should be topic neutral. "Designating very widely used terms as logical constants has the effect of promoting further generality in logic. Argument forms depend on logical constants, and choosing ubiquitous terms as logical constant ensures that non-trivial arguments will be ubiquitous. This in turn ensures that logic will be widely applicable."[21]

(E) The generality requirement seems, however, to undercut (E). *Prima facie*, the generality criterion makes Kripkean models relevant to establishing the logical contingency of $\ulcorner \Diamond_{S5}\alpha \urcorner$ when '$\Diamond_{S5}$' is the logical possibility operator. To paraphrase Hanson, the proponent of Kripkean semantics is willing to accept as counterexamples to $\ulcorner \Diamond_{S5}\alpha \urcorner$ not only full-fledged ways things might have been but also fragments of such ways. Doing so can be seen as introducing further generality into S5 modal logic, a generality that comes from taking S5 modal logic to be applicable even to the tiniest and most bizarrely gerrymandered fragments of logical modal reality. I don't know how plausibly to weaken the generality requirement in a way that provides a rationale for both (C) and (E).

One way to try to maintain (E) is to follow Hanson and weaken the generality criterion by merely requiring that there be topic-neutral terms included in the class of logical terminology (Hanson (1997) p. 376). The fixed modal operator semanticist is interested in doing the logic of logical possibility and so desires to treat '$\Diamond_{S5}$' as a logical constant and fixes its meaning from one model to another in terms of the collection of logically possible worlds rather than in terms of other collections of possible worlds (the physically possible ones, the possible worlds which contain George W. Bush, etc.). This is consistent with maintaining the weakened generality criterion, as long as we grant a healthy dose of topic-neutral terms in our logical vocabulary such as the Boolean connectives, the identity symbol, and first-order quantifiers. The problem is that satisfying the generality criterion so weakened doesn't guarantee that an account of logical possibility in first-order logic will reflect (C). For example, an account of logical possibility that treats the Boolean connectives and the identity symbol as logical constants but fixes the meaning of the quantifiers in terms of the totality of the world's individuals seems to satisfy the weakened generality requirement, but it does not satisfy (C).[22] In sum, according to this non-modal account of logical possibility, the selection of logical terms must ensure that logic exhibits an appropriate generality. But the notion of *appropriate generality* is vague, and I don't know of a non-question-begging way of making it precise so that both (C) and (E) hold.

(III) *Possible Meaning (PM) Account of Logical Possibility*

(C) Relative to an ordinary first-order language, a logically possible state of affairs in which there are just *n* individuals for finite *n* is a possible meaning (or use) for the non-logical elements of the language,[23] e.g., a possible use for the first-order quantifiers according to which they range over *n* individuals. We appeal to genuine possibilities to underwrite domain restrictions in first-order logic.[24] For each positive integer *n*, a possible use of the quantifiers is to range over just *n-1* of the things that, in fact, exist. No first-order quantificational sentence should count as a logical truth if its truth depends on one use of the quantifiers that occur in it.

(D) The PM approach is compatible with (D), for in assigning a truth-value to a quantificational sentence relative to a subset of worldly individuals, we are not considering different ways earth, air, fire, and water could have been distributed. Rather, we represent one way the quantifiers of a language can be used to range over what, in fact, exists.[25] For example, we needn't think that the only way for the sentence, ‘Everybody is large’ to be true while keeping the meaning of ‘large’ fixed is for it to be interpreted in a merely possible world whose residents are all mesomorphs. Rather we imagine a context in which an utterance of the sentence would make it actually true. For example, while looking at a picture of NFL football players, I say, “Everybody is large”. The context here determines that the quantifier ‘everybody’ ranges over just those pictured and not all human beings. Clearly, ‘Everybody is large’ has at least as many uses as there are domains for ‘Everybody’.[26] What do we get when we extend this approach to modal logic?

(E) A logically possible state of affairs that restricts the collection of logically possible worlds represents a possible use for the S5 diamond and box. For example, corresponding to the classification of ‘$\Diamond_{S5}$’ as a modal operator is a well-defined semantic role for the diamond that fixes a range of possible readings. Giving a specification of an S5 first-order model structure (i.e. spelling out the details required by (1)-(5) on p. 2) is the selection of one reading for ‘$\Diamond_{S5}$’ from the range of possible S5 readings. No modal sentence should count as a logical truth if its truth depends on one use of the S5 modal operator(s) that occur in it. So, ‘it is logically possible that’ should not be treated as a logical term because if we do so, we do fail to capture the idea that a logically true modal sentence remains true on each way of understanding the sentence that is consistent with the semantic functioning of the modal “quantifier” ‘$\Diamond_{S5}$’.[27] I don’t see a promising motive for making possible uses for first-order quantifers but not S5 modal operators (understood semantically as quantifiers) relevant to what is logically possible. The *(PM) Approach* doesn’t seem to support (E).

## 6. **Conclusion**

The criticism of my argument against (TLP) requires a rationale for allowing the universal and existential quantifier to range over various collections of individuals in determining logical truth for an interpreted non-modal first-order language, but not allowing the modal operators to range over the various collections of possible worlds in determining logical truth for an interpreted modal language. I have conceived of the required rationale as an account of logical possibility that reflects (C)-(E).

(C) For each natural number *n* there is a logically possible state of affairs according to which there are just *n* individuals.

(D) It is metaphysically necessary that there are at least infinitely many individuals.

(E) There is no logically possible state of affairs according to which there are fewer logically possible worlds than there are.

I am not a logical primitivist. I, along with others, think that there is a story to tell about logical possibility. I don't know of a plausible non-primitivist account of logical possibility that reflects (C)-(E). Consequently, since I accept (C) and (D), I think that maintaining (E) is problematic. Given the treatment of the modal operators as quantifiers over possible worlds, there is no plausible non-primitivist account of logical possibility which justifies regarding them in a way that is essentially different from the standard semantic treatment of the quantifiers of first-order logic. Since the range of '$\forall x$' and '$\exists x$' may vary from interpretation to interpretation, this, by analogy, motivates the corresponding variability in the interpretation of the modal operators, i.e., the ranges of the modal operators can be *any* non-empty subset of the totality of possible worlds. This in turn justifies the feature of universal validity according to which what is logically true in a first-order modal language with S5 modal operators is true at **G** of *every (non-empty) subset* of the set **K** of possible worlds in an S5-model structure.

The question arises, what, exactly is the modal status of true statements about what is logically possible? This is a complex question and the defender of the Kripkean account has to have some story to tell here. Specifically, given that statements about what is logically possible are not logically necessary, in what sense are they necessarily true? My response runs roughly as follows.

I am a proponent of the model-theoretic conception of the logical properties. With respect to a language adequate for logical modality, the S5 diamond may be read as 'there exists a model of' (turning the modal language L into a formalized version of the modal meta-language for first-order logic). True sentences of the form ⌜$\Diamond_{S5}\alpha$⌝ are classical model-theoretic truths and are, therefore, necessarily true. The likely candidate for the modality is mathematical,[28] in which case knowledge of these model-theoretic truths is of a mathematical sort. For example, to know that each instance of $\Diamond_{S5}$ *at least n things exist* is true ($n \geq 1$), requires that I know that an infinite model exists. Since such a model is a mathematical entity, logic does not take an absolute epistemic priority over all mathematical knowledge. There is an interaction between logical knowledge and knowledge of mathematics (i.e., knowledge of set theory). Moreover, on this approach, the necessity of, say, (B) '$\Diamond_{S5}\exists x\exists y(x\neq y)$' boils down to the necessary existence of the domain of mathematical entities (e.g., pure sets) over which the first-order existential quantifiers may range. This perspective of the necessity of true statements about logical possibility is reflected in the explanations of first-order logical truth given by, among others, Putnam, Resnik, Maddy, and Shapiro.[29] In short, the requirement that a logically true modal sentence remain true on all non-empty restrictions of the set **K** of logically possible worlds does not rule preclude a substantive account of the necessity of true statements about what is logically possible.

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[1] A fundamental intuition underlying Kanger's work in the logic of the logical modalities suggests how this may be put together with the view that the box and diamond attach to statements and not to names of statements. Following the discussion in Lindström ((1998) p. 214), Kanger's basic idea is of a metalinguistic interpretation of the modal operator that can be expressed in the following reflection principle:

(R) $S \models$ ⌜$O\alpha$⌝ iff $P(\alpha, S)$

Where 'O' is an operator of the object language, 'P' a metalinguistic predicate that may be satisfied by a formula $\alpha$ and an interpretation S. If we think of (R) as giving not only the truth conditions for ⌜$O\alpha$⌝, but also as explaining the meaning of the operator O in terms of the meta-linguistic predicate, then (R) provides a metalinguistic interpretation of O. Correspondingly, the modal operator 'it is logically necessary that' of the object language L is obtained from L's meta-language by "reflecting" the corresponding meta-linguistic predicate. On a Tarskian model-theoretic understanding of logical necessity, one instantiation of (R) is

$\ulcorner \Box_{S5}\alpha \urcorner$ is true iff $\alpha$ is true in every interpretation of the object language,

where the right side of the biconditional may be spelled out formally in the semantic clause for '$\Box_{S5}$'. Accordingly, it is logically necessary that $\exists x(x=x)$ (according to classical logic) because the sentence '$\exists x(x=x)$' is logically necessary, i.e., true in every interpretation of the object language.

[2] Since $\Phi_M$(Female(x) & U.S. President(x),**G**)=**F** relative to the assignment of $\mathbf{u_1}\in\Psi(\mathbf{G})$ to 'x', $\Phi_M$($\exists$x(Female(x) & U.S. President(x)),**G**)=**F**. Since **M***: **K**={**G**}, $\Phi_M$($\Diamond_{S5}\exists$x(Female(x) & U.S. President(x)),**G**)=**F**, i.e., (A) is false in **M**. Here's why (B) is also false in **M**. $\Phi_M(\exists x\exists y(x\neq y),\mathbf{G})=\mathbf{F}$ since $\Phi_M((x\neq y),\mathbf{G})=\mathbf{F}$ relative to the assignment of $\mathbf{u_1}\in\Psi(\mathbf{G})$ to 'x' and 'y' and $\mathbf{u_1}$ is the only object in the domain of **G**. Therefore, $\Phi_M(\Diamond_{S5}\exists x\exists y(x\neq y),\mathbf{G})=\mathbf{F}$ because **M***: **K**={**G**}.

[3] The only constraint on non-empty subsets $\mathbf{K_i}$ is that the actual-world element **G** of a model structure must be in the set $\mathbf{K_i}$ of worlds.

[4] See, for example, Cocchiarella (1975a) p. 13, Hintikka (1980) pp. 283-284, and Hanson and Hawthorne (1985) p. 9.

[5] With respect to languages adequate for representing logical modalities, there are well-known fixed modal operator accounts. Of course, fixed modal operator accounts can and do differ. The relevant feature they share is that the space of possibilities correlated with the domain of the modal operators, whether the possibilities be construed as state descriptions, model sets, possible worlds, etc., is fixed in the determination of logical truth. In Kripkean terms, on a given interpretation of the modal operators, the non-Kripkean account equates *truth in a structure* with logical truth, while the Kripkean account equates *universal validity* with logical truth. Hence, less comes out logically true on the Kripkean account. What I am calling the fixed modal

operator and Kripkean accounts of logical truth Cocchiarella and Hintikka have called primary and secondary semantics, and standard and non-standard semantics, respectively.

[6] See Burgess (1999) pp. 86-87, who makes a similar point in defense of *universal validity* in S5 modal propositional logic.

[7] Hintikka (1982) p. 381.

[8] This formulation is due to Field (1991) p. 9.

[9] Pollock (1966) p. 131.

[10] The semantic treatment of the S5 operators as existential and universal quantifiers over possible worlds will not result in L sentences containing free modal variables (i.e., variables ranging over **K**), and so we simplify and eliminate in (LFP') talk of uniform restrictions of free modal variables.

[11] Note that the informal counterexample to (A) here is secured without having to restrict the range of '$\Diamond_{S5}$'; the modal quantifier in the false sentence '$\Diamond_{S5}\exists x$ (Female(x) & Non-female(x))' ranges over nothing less than the totality of logically possible worlds. This is in contrast to the formal counterexample to (A) generated earlier (on p. 3), which is based on a model structure containing just one possible world. Hence, the legitimacy of the informal counterexample to (A) cannot be questioned because of a restriction on the totality of logically possible worlds.

In general, no appeal to a restriction of the totality of logically possible worlds need be made in order to establish that a logically contingent modal formula $\alpha$ with either sentential or predicate variables (e.g., '$\Diamond_{S5}\alpha$', '$\Diamond_{S5}\exists x Fx$') is logically contingent, i.e., is not true in virtue of form. There are many sentences that are impossible and many predicates that could not possibly be instantiated that can be used to replace the non-logical terminology of a logically contingent true sentence in order to demonstrate that the sentence is indeed logically contingent. At the level of modal propositional logic an extensionally correct account of formal, logical truth does not require restrictions on the domain of logically possible worlds. Taking '$\Diamond_{S5}$' to range over nothing less than the totality of logically possible worlds, we may establish that, say, ⌜$\Diamond_{S5}\alpha$⌝ and ⌜$\Box_{S5}\Diamond_{S5}\alpha$⌝ are not the forms of logical truths by substituting logical falsehoods for $\alpha$. For example, ⌜ $\Diamond_{S5}\alpha$⌝ (where '$\alpha$' is a sentential variable) has a false instance since '$\Diamond_{S5}$snow is white and snow is not white' is true at no world (substituting 'snow is white and snow is not white' for '$\alpha$'). The sentence is false interpreting '$\Diamond_{S5}$' as ranging over the totality of logically possible worlds.

In contrast to modal propositional logic, at the level of first-order modal logic there are sentences that contain only logical terminology such as '$\Diamond_{S5}\exists x\exists y(x\neq y)$' and '$\Diamond_{S5}\exists x\exists y\forall z(x\neq y\&(z=x\vee z=y))$'. Since such sentences have no sentential or predicate variables for which we may substitute, in order to establish that the sentences are not true in virtue of form we *must* appeal to domain restrictions on the set of logically possible worlds. It appears that at the level of first-order modal logic, but not at the level of modal propositional logic, an appeal to domain restrictions on the set of logically possible worlds is necessary in order to fix the extension of formal, logical truth. Since what is at issue is the feature of *universal validity* according to which the ranges of the modal operators vary from model to model, I have chosen to defend the Kripke account and pursue criticism of (TLP) at the level of first-order modal logic where the rationale for domain restrictions on the collection of logically possible worlds is clearest. Also, some of the discontents (e.g., Field (1989)) pursue their criticism of the Kripke account at the level of first-order modal logic, and it is useful to develop a response at the same level. I have, however, steered clear of issues in first-order modal logic that are not relevant to the concerns of this paper such as the meaningfulness of *de re* quantification into contexts of logical modality.

[12] Hazen (1999) also distinguishes between "truths of logic" (i.e., truths which it is the business of logicians *qua* logicians to discover) and "logical truths" (i.e., such a proposition makes any argument valid when it is the conclusion) (p. 79). I am in agreement with Hazen's claim that it is

highly unlikely that the two are coextensive. So just because true statements about what is logically possible in first-order logic are *truths of logic* as I believe, it is not obvious that they thereby qualify as *logical truths*.

[13] To say that a sentence is true in a logically possible world *w* is to say that had *w* been the actual world, that sentence would have been true.

[14] To say that a modal sentence S is true in a logically possible modal reality R is to say had R been the case, S would have been true.

[15] For example, see McFetridge (1990), Hale (1996), and Field (1989). Here's a motive for logical primitivism derived from (C), (D), and the semantics of first-order quantifiers. By (C), there are logically possible states of affairs according to which there are just *n* individuals for each natural number *n*. Because first-order quantifiers are logical constants, there is no non-logical element of their meaning (this is rejected on the Possible Meaning Approach to logical possibility, see note 25 below). For example, '∀' must be interpreted as 'for all the world's individuals' from one first-order model to another. So from one logically possible situation to another the first-order quantifiers range over all of the world's individuals, and different quantifier ranges must be taken to represent the world with different numbers of individuals. Given (D), these must be ways the world could—in a logical sense—be.

[16] For example, see Fine (1994) and (2002), Hanson (1997), and Shapiro (1993). For criticism of Hale who makes logical possibility basic see Shalkowski (2004).

[17] Shapiro (1993) p. 465.

[18] This account is in Hanson (1997) and (2002).

[19] Hanson (1997) p. 390.

[20] Hanson (1997) p. 388.

[21] Hanson (1997) p. 375.

[22] For explication of such an account of logical possibility and a defense of making first-order sentences of the form '*n* individuals exist' logical truths (for each natural number *n*), see Almog (1989) and Williamson (1999). According to such an account, it is logically impossible for there to be finitely many first-order individuals and so quantifier domain restrictions are ruled out. The strategy of attacking the first premise of my argument against (TLP) given above (p.8) in defense of the Kripkean account by questioning the standard account of how quantifiers work in first-order logic is not considered in this paper. It is standard, of course, to think that a first-order quantification is a logical truth only if it remains true no matter how the quantifiers are restricted (and no matter how the interpretations of non-logical expressions are varied).

[23] To understand logical consequence we must understand how it is possible for sentences to have truth-values other than the ones they actually have. For example, if the conclusion of an invalid argument is *Bill Clinton is a Democrat* we must think that this sentence could (logically) be false. How could *Bill Clinton is a Democrat* be false? Since truth depends both on the use of words and the way the world is, there are two ways of understanding possible truth and falsity

We can say that *Bill Clinton is a Democrat* could be false if the words *Bill Clinton*, and *is a Democrat* are used ordinarily but the world were different (imagine that Bill had different life experiences which made him into a Republican). Alternatively, *Bill Clinton is a Democrat* could be false if the world is as it is but the component words were used differently (imagine that *is a Democrat* means *is a bachelor*). This second way of changing a sentence's truth-value leads to the notion of logical possibility appealed to in the PM Approach to logical possibility. The resulting characterization of first-order logical truth is: $\alpha$ is a logical truth iff there is no possible use or meaning of its constituent non-logical elements under which $\alpha$ is false.

[24] Like the Logical Primitivist approach but unlike the Non-modal approach, the Possible Meaning approach to logical possibility establishes (A) by appealing to legitimate possibilities, but like the Non-modal Approach and unlike the Logical Primitivist Approach, the Possible

Meaning approach does not appeal to *logically possible ways the world could be* in establishing (A).

[25] I don't believe that the conception of the universal quantifier as a logical constant is seriously threatened by the PM approach. In fact, its treatment of the quantifiers parallels the model-theoretic treatment. For example, Hodges tells us that in the first-order languages of model theory, the quantifier symbols "…always mean 'for all individuals' and 'for some individuals'; but what counts as an individual depends on how we are applying the language. To understand them, we have to supply a *domain of quantification* [italics are Hodges']" Hodges (1986, p.144). Correspondingly, a possible use for the quantifier symbols of a language is L determined by a particular domain of discourse for L. As L may have different domains of discourse (i.e. may have different applications), there are different uses for the quantifier symbols of L. The fact that the quantifiers are logical constants means that the part of their meaning that is fixed is the portion of the domain they pick out (e.g., the existential quantifier refers to at least one member of the domain and the universal quantifier refers to all of it), and the part of their meaning that can be varied (i.e. the non-logical element of the quantifiers) in order to make a sentence true or false is the size of the domain. So, the PM theorist may regard the existential quantifier as a logical constant and still think that a sentence like '$\exists x \exists y(x \neq y)$' should be logically contingent since a possible use of the variables is to range over exactly one thing, and when the sentence is used this way it is false.

[26] That the context determines that 'everybody' means *all those pictured* and not *everybody in the universe* is controversial. For example, according to Bach (2000), the utterance 'Everybody is large' semantically expresses the proposition that everybody in the universe is large, and conveys as—what he calls—an impliciture the proposition that everyone in the picture is large. Contrary to the story I tell, on Bach's view the context does not contribute to the meaning of 'Everybody', and what is literally meant by 'Everybody is large' (which is false on Bach construal of the

meaning of 'Everybody') is not what the speaker intends to convey. The claims made in my paper that a possible use of an ordinary language quantifier is a possible meaning for it and that a use (and, therefore, meaning) of an ordinary language quantifier is determined by context puts me in the camp with those who, against Bach, believe in some kind of context-dependent domain restrictions on ordinary language quantifiers, e.g. Neale (1990), and Stanley & Szabo (2000) who differ on how the restriction is expressed in a quantification—either with some kind of tacit syntactic element or with a semantic element.

[27] On an interpretation of '$\Box_{S5}$' in (F) that makes it range over the entire collection of logically possible worlds, (F) is a true.

$$\text{(F) } \Box_{S5}\exists x\exists y(x\neq y)\rightarrow\Box_{S5}\exists x\exists y\exists z((x\neq y)\&(y\neq z)\&(x\neq z))$$

This suffices to make (F) a logical truth on the fixed modal operator account of logical truth, since the antecedent is not, and, therefore, cannot be true. According to the PM approach, we may appeal to a possible use of '$\Box_{S5}$' in order to make the antecedent true in showing that the negation of (F) is logically possible. For example, consider the use of '$\Box_{S5}$' according to which it ranges over all logically possible worlds in which there are at least two individuals (used this way '$\Box_{S5}$' means *necessary relative to the fact that there are at least two individuals*). (F) is false on this use of the S5 box for it is *necessary relative to the fact that there are at least two individuals* that there are at least two individuals, but it is not *necessary relative to the fact that there are at least two individuals* that there are at least three. This is so because among all the possible worlds in which there are at least two individuals we suppose that there are just two individuals, hence not at least three.

[28] As suggested by, among others, Parsons (1986), p. 373-374.

[29] For discussion see Shapiro (1993), p. 455.

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